Finite Math - Spring 2017

Lecture Notes - 3/6/2017

Homework

- Section 4.1 - 1, 5, 7, 9, 10, 11, 12, 13, 17, 20, 21, 23, 25, 26, 27, 28, 31, 33, 65, 68, 69, 71, 75, 80

Section 4.1 - Systems of Linear Equations in Two Variables

Suppose we go to a movie theater and there are two packages for discounted tickets:
Package 1: 2 adult tickets and 1 child ticket for \$32
Package 2: 1 adult ticket and 3 child tickets for \$36

Based off of this information, can we figure how much the adult and child ticket discount prices are?

We can! To do this let A stand for the price of the adult ticket and let C stand for the price of the child ticket, then we get the following two equations from the two packages:

$$\begin{array}{rcrcl} 2A & + & C & = & 32 \\ A & + & 3C & = & 36 \end{array}$$

This is a system of two linear equations in two variables. To find the answer, we need to find a pair of numbers (A, C) which satisfy *both* equations simultaneously.

Definition 1 (System of Two Linear Equations in Two Variables)**.** *Given the* linear system

$$\begin{array}{rcrcl} ax & + & by & = & h \\ cx & + & dy & = & k \end{array}$$

where a, b, c, d, h, and k are real constants, a pair of numbers $x = x_0$ and $y = y_0$ (often written as an ordered pair (x_0, y_0)) is a solution *of this system if each equation is satisfied by the pair. The set of all such ordered pairs is called the* solution set *for the system. To* solve *a system is to find its solution set.*

There are a few ways we can go about solving this: *graphically*, using *substitution*, and *elimination by addition*.

Solving by Graphing. To solve this problem by graphing, we simply graph the two equations in the system, then find the intersection. Since we're relying on a graph to find this point, we need to check our solution in the equations of the system.

The blue line is the graph of $2A + C = 32$ and the purple line is the graph of $A + 3C = 36$. The red point is the intersection point $(12, 8)$. So the ticket prices are \$12 for an adult ticket and \$8 for a child ticket. Check the point $(12, 8)$ in both equations:

$$2A + C = 2(12) + 8 = 24 + 8 = 32 \quad \checkmark$$

and

$$A + 3C = 12 + 3(8) = 12 + 24 = 36 \quad \checkmark.$$

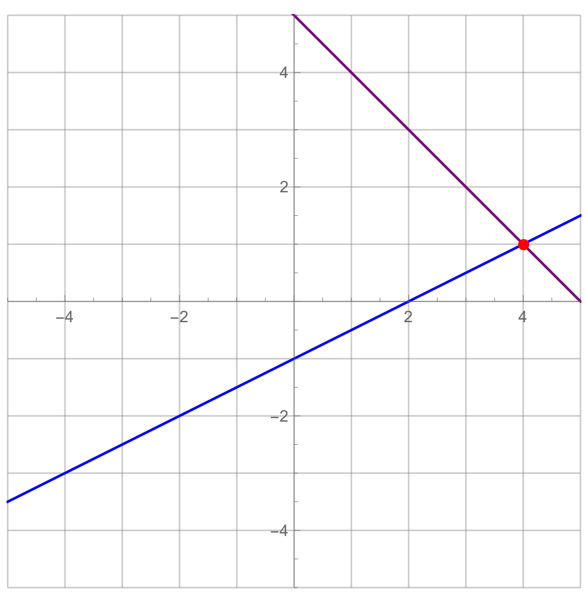
This verifies the solution.

There are actually 3 types of solutions to a system of linear equations

(1) Consider the system

$$\begin{array}{rcrcl} x & - & 2y & = & 2 \\ x & + & y & = & 5 \end{array}$$

If we graph the lines, we get

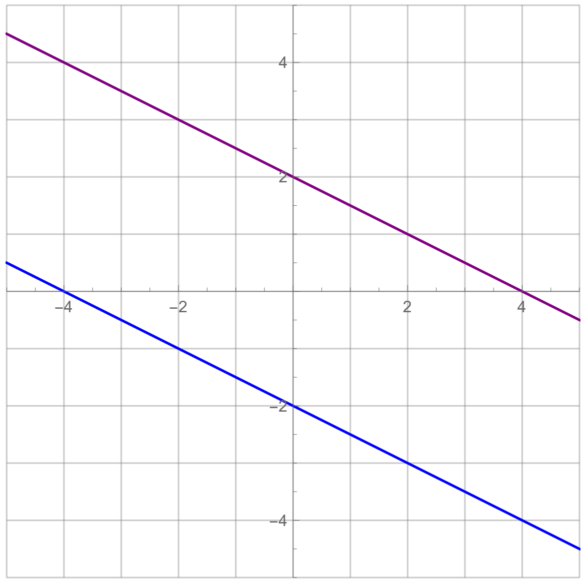


In this case, like before, we see only the *one solution* at $(4, 1)$. (You should check this in the system!)

(2) Consider the system

$$\begin{array}{rcrcr} x & + & 2y & = & 4 \\ 2x & + & 4y & = & 8 \end{array}$$

If we graph the lines, we get

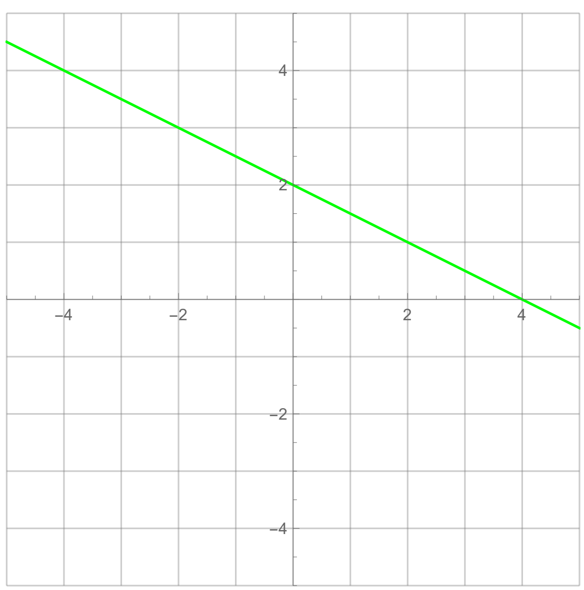


In this case, the lines are parallel and so they never intersect. In this case, there is *no solution*.

(3) Consider the system

$$\begin{array}{rcrcr} 2x & + & 4y & = & 8 \\ x & + & 2y & = & 4 \end{array}$$

If we graph the lines, we get



Here, both of the lines are exactly the same. In this case, there is an infinite number of solutions.

Definition 2. *A system of linear equations is called* consistent *if it has one or more solutions and* inconsistent *if it has no solutions. Further, a consistent system is called* independent *if it has exactly one solution (called the* unique solution*) and is called* dependent *if it has more than one solution. Two systems of equations are called* equivalent *if they have the same solution set.*

Theorem 1. *The linear system*

$$\begin{array}{rcrcl} ax & + & by & = & h \\ cx & + & dy & = & k \end{array}$$

must have

(1) *Exactly one solution (consistentent and independent).*

(2) *No solution (inconsistent).*

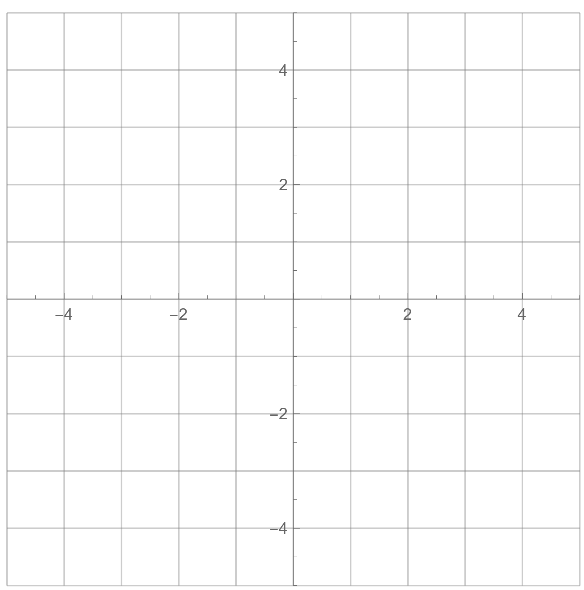
(3) *Infinitely many solutions (consistent and dependent).*

There are no other possibilities.

Example 1. *Solve the following systems of equations using the graphing method. Determine whether there is one solution, no solutions, or infinitely many solutions. If there is one solution, give the solution.*

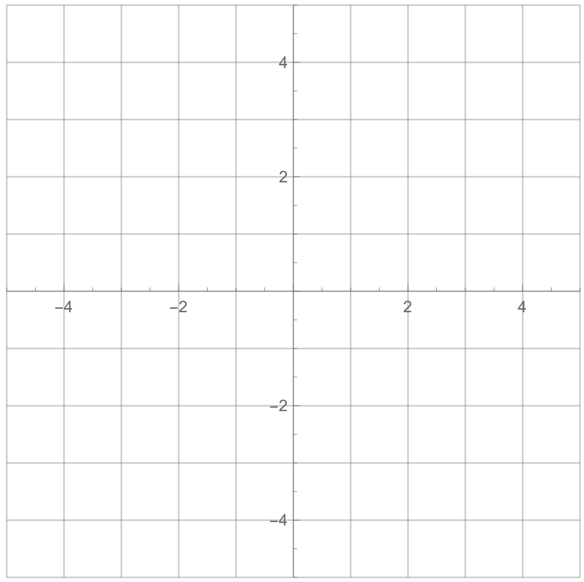
(a)

$$\begin{array}{rcrcl} x & + & y & = & 4 \\ 2x & - & y & = & 5 \end{array}$$



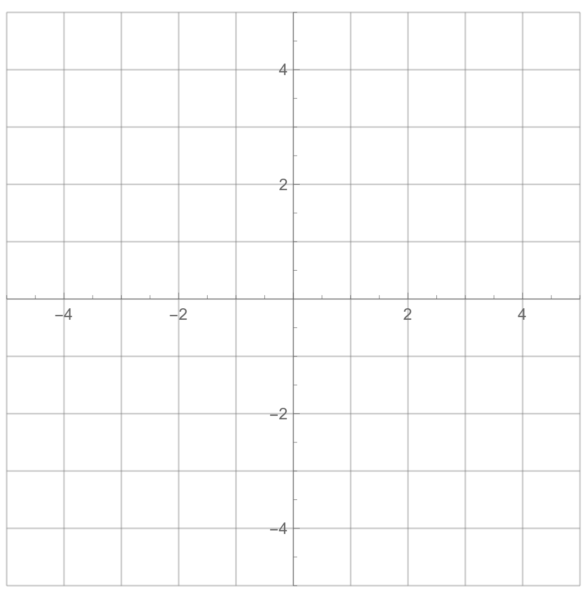
(b)

$$\begin{array}{rcrcl} 6x & - & 3y & = & 9 \\ 2x & - & y & = & 3 \end{array}$$



(c)

$$\begin{array}{rcrcl} 2x & - & y & = & 4 \\ 6x & - & 3y & = & -18 \end{array}$$



Solving by Substitution. When solving a system by substitution, we solve for one of the variables in one of the equations, then plug that variable into the other equation.

Example 2. *Solve the following system using substitution*

$$\begin{array}{rcrcl} 2x & - & y & = & 3 \\ x & + & 2y & = & 14 \end{array}$$

Solution. *Let's solve the first equation for y. To do this, we'll move the y to the right side, and the 3 to the left:*

$$2x - 3 = y$$

Then we plug this into the second equation for y:

$$x + 2(2x - 3) = 14$$

then we solve for x in this

$$x + 4x - 6 = 5x - 6 = 14$$

which gives

$$5x = 20$$

and so

$$x = 4.$$

Then we plug this into the equation we have for y to find that

$$y = 2(4) - 3 = 8 - 3 = 5$$

And so the solutions is $x = 4, y = 5$.

Example 3. *Solve the following system using substitution*

$$\begin{array}{rcrcl} 3x & + & 2y & = & -2 \\ 2x & - & y & = & -6 \end{array}$$

Solution. $x = -2, y = 2$

Solving Using Elimination. We now turn to a method that, unlike graphing and substitution, is generalizable to systems with more than two variables easily. There are a set of rules to follow when doing this

Theorem 2. *A system of linear equations is transformed into an equivalent system if*

(1) *two equations are interchanged*

(2) *an equation is multiplied by a nonzero constant*

(3) *a constant multiple of one equation is added to another equation.*

Example 4. *Solve the following system using elimination*

$$\begin{array}{rcrcr} 3x & - & 2y & = & 8 \\ 2x & + & 5y & = & -1 \end{array}$$

Solution. *If we subtract the second equation from the first one, we end up with the new system*

$$\begin{array}{rcrcr} x & - & 7y & = & 9 \\ 2x & + & 5y & = & -1 \end{array}$$

Now, we can subtract 2 times the first equation ($2x - 14y = 18$) from the second equation to get

$$\begin{array}{rcrcr} x & - & 7y & = & 9 \\ & & 19y & = & -19 \end{array}$$

Now we divide the second equation by 19 to get

$$\begin{array}{rcrcr} x & - & 7y & = & 9 \\ & & y & = & -1 \end{array}$$

and finally, we will add 7 times the second equation ($7y = -7$) to the first equation

$$\begin{array}{rcrcr} x & & & = & 2 \\ & & y & = & -1 \end{array}$$

This gives the answer of $x = 2, y = -1$.

We could have also used a combination of substitution and elimination above, for example, once we knew that $y = -1$, we could have just plugged that into the first equation, but this solution was a little preview for the later sections.

Example 5. *Solve the system using elimination*

$$\begin{array}{rcrcr} 5x & - & 2y & = & 12 \\ 2x & + & 3y & = & 1 \end{array}$$

Solution. $x = 2, y = -1$